# Chapter 10 – 1, 2 Introduction to Limits and Continuity

### Limits

Conceptually, a function f(x) has a limit L as x approaches some value c if the value of f(x) gets closer and closer to L as x gets closer and closer to c (from either direction). If that is true then we write



This is referred to as a two-sided or unrestricted limit.

A left-hand limit is the limit of f(x) as x approaches c from the left (i.e., through values that are always less than c):



A right-hand limit is the limit of f(x) as x approaches c from the right (i.e., through values that are always greater than c):



### Theorem on Existence of a Limit

A two-sided limit exists if and only if the left-hand and right-hand limits exist and are equal:



### Continuity

Conceptually, a function f(x) is continuous on the interval (a, b) if you can draw the graph of f(x) on the interval (a,b) without lifting the pen from the paper. If there any gaps or abrupt changes in the function then it would be necessary to lift the pen to span the gap or the abrupt change and we say the function is discontinuous.

### Formal Definition of Continuity

A function f(x) is continuous at x = c if $\lim\_{x\to c}f(x)$ exists, $f(c)$ exists, and $\lim\_{x\to c}f\left(x\right)=f(c)$. A function f(x) is continuous on the interval (a,b) if f(x) is continuous at every value of x in the interval.

**Example 1.** 

The function $f(x) = 2x+3$ is continuous at x =3 because:



**Example 2.** Find . This function does not have a limit as x approaches 0 because the two one-sided limits have different values.

This function is not continuous at x=0 because the limit as x approaches 0 is undefined (does not exist).

## Properties of Limits

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**Example 3.** 

This function is continuous at x = 1 because the limit as x approaches 1 is equal to the value of the function at x = 1.

**Example 4.**  is undefined.

Find 

This function is not continuous at x = 0 because the limit as x approaches 0 is undefined.

This function is not continuous at x = 2 because the function is undefined at x = 2.

**Example 5.** Find 

The function is not continuous at x = -2 because the function is undefined at x = 2.

**Example 6.** Find 



The slope of the line that is tangent to the graph of f(x) = x2 at any point (x, x2) is 2x.

**Example 7.** An over-night delivery service charges a flat rate of $15.00 to ship a parcel weighing up to 2 pounds and a surcharge of $5.00 for each additional pound (or fraction thereof).

(A) Write a piecewise definition of the shipping cost C(x) for a parcel weighing x pounds.

Floor function: $\left⌊x\right⌋=$ the largest integer less than or equal to x. This is also known as the greatest integer function.

Ceiling function: $\left⌈x\right⌉=$ the smallest integer greater than or equal to x.



(B) Graph C(x) for 0 < x ≤ 8.

(C)

$$\lim\_{x\to 5^{-}}C\left(x\right)=30$$

$$\lim\_{x\to 5^{+}}C\left(x\right)=35$$

$$\lim\_{x\to 5}C\left(x\right) is undefined because the one-sided limits are not equal.$$

C(x) is not continuous at x = 5 because the limit of C(x) as x approaches 5 is undefined.