# Chapter 10 – 6b Differentials

**Example 5.** Given the function $y=\left(1-2x\right)\sqrt[3]{x^{2}}$, find dy and ∆y when x = 8 and ∆x = 0.01.

$$y=f\left(x\right)=\left(1-2x\right)x^{{2}/{3}}=x^{{2}/{3}}-2x^{{5}/{3}}$$

$$dy=f^{'}\left(x\right)dx=\left(\frac{2}{3}x^{-{1}/{3}}-\frac{10}{3}x^{{2}/{3}}\right)dx=\left(\frac{2}{3}8^{-{1}/{3}}-\frac{10}{3}8^{{2}/{3}}\right)0.01=\left(\frac{1}{3}-\frac{40}{3}\right)0.01=-0.13$$

$$∆y=f\left(x+∆x\right)-f\left(x\right)=\left(1-2∙8.01\right)\sqrt[3]{8.01^{2}}-\left(1-2∙8\right)\sqrt[3]{8^{2}}≈-60.13006-\left(-60\right)=-0.13006$$

**Example 6.** From past records, it is estimated that a company will sell N units of a product after spending x thousands of dollars on advertising:

$$N\left(x\right)=60x-x^{2}⟹N^{'}(x)=60-2x$$

A) Use the differential of y to approximate the increase in sales if the advertising budget increases from $10,000 to $11,000. Compare your approximation to the actual value.

$$x=10 and ∆x=dy=1$$

$$dy=N^{'}(x)dx=\left(60-2x\right)dx=\left(60-2∙10\right)1=40 units$$

$$∆y=N\left(x+∆x\right)-N\left(x\right)=N\left(11\right)-N\left(10\right)=539-500=39 units$$

B) Use the differential of y to approximate the increase in sales if the advertising budget decreases from $21,000 to $20,000. Compare your approximation to the actual value.

$$x=21 and ∆x=dy=-1$$

$$dy=N^{'}\left(x\right)dx=\left(60-2x\right)dx=\left(60-2∙21\right)\left(-1\right)=-18 units$$

$$∆y=N\left(x+∆x\right)-N\left(x\right)=N\left(20\right)-N\left(21\right)=800-819=-19 units$$

**Example 7.** For a company that manufactures tennis rackets, the average cost per racket $\overbar{C}$ is found to be

$$\overbar{C}\left(x\right)=\frac{400}{x}+5+\frac{1}{2}x, x\geq 1$$

where x is the number of rackets produced per hour.

$$\overbar{C}\left(x\right)=400x^{-1}+5+0.5x ⟹ \overbar{C}^{'}\left(x\right)=-400x^{-2}+0.5=-\frac{400}{x^{2}}+\frac{1}{2}$$

A) Use the differential of $\overbar{C}\left(x\right)$ to approximate the change in the average cost per racket if production increases from 20 to 25 rackets per hour. Compare your approximation to the actual value.

$$dy=\overbar{C}^{'}\left(x\right)dx=\left(-\frac{400}{x^{2}}+\frac{1}{2}\right)dx=\left(-\frac{400}{20^{2}}+\frac{1}{2}\right)5=-\$2.50$$

$$∆y=\overbar{C}\left(25\right)-\overbar{C}\left(20\right)=\left(\frac{400}{25}+5+\frac{1}{2}25\right)-\left(\frac{400}{20}+5+\frac{1}{2}20\right)=33.50-35.00=-\$1.50$$

B) Use the differential of $\overbar{C}\left(x\right)$ to approximate the change in the average cost per racket if production decreases from 20 to 18 rackets per hour. Compare your approximation to the actual value.

$$dy=\overbar{C}^{'}\left(x\right)dx=\left(-\frac{400}{x^{2}}+\frac{1}{2}\right)dx=\left(-\frac{400}{20^{2}}+\frac{1}{2}\right)\left(-2\right)=\$1.00$$

$$∆y=\overbar{C}\left(18\right)-\overbar{C}\left(20\right)=\left(\frac{400}{18}+5+\frac{1}{2}18\right)-\left(\frac{400}{20}+5+\frac{1}{2}20\right)=36.22-35.00=\$1.22$$

Notice that the estimate provided by the differential dy is not very accurate because dx is quite large (5 and -2 respectively).