# Chapter 12 – 6a Optimization

**Example 1.** How would you divide a 10-inch line so that the product of the two lengths is a maximum?

$$f\left(x\right)=x\left(10-x\right)=10x-x^{2}$$

$f^{'}\left(x\right)=10-2x=0$ means the slope is 0 at $x=5$

$f^{''}\left(x\right)=-2$ means that $f(x)$ is always concave down.

The maximum product is $f\left(5\right)=25$ when the line is divided exactly in half.

**Example 2.** A homeowner wants to enclose a rectangular 400 sq ft play area with a fence that costs $3.25 per linear foot. Find the dimensions of the play area that will minimize the cost of the fencing. What is the minimum cost?

$$C\left(x\right)=3.25\left(2x+2\frac{400}{x}\right)=6.5x+\frac{2600}{x}, x>0$$

$C^{'}\left(x\right)=6.5-\frac{2600}{x^{2}}=0$ at $x=20.$

$C^{''}\left(x\right)=\frac{5200}{x^{3}}>0$ on the domain of this function so the function is always concave up.

The minimum cost is $f\left(20\right)=\$260$ when the play area is a 20 by 20 square.

**Example 3.** The homeowner in the previous problem can spend $390 on fencing for a rectangular play area. Find the dimensions of the play area that will yield the largest area. What is the largest area?

$$A\left(x\right)=x\left(60-x\right)=60x-x^{2}, x>0$$

$A^{'}\left(x\right)=60-2x=0$ at $x=30$

$A^{''}\left(x\right)=-2$ means the function is always concave down.

The maximum area is $A\left(30\right)=900$ sq ft when the play area is 30 ft by 30 ft.

**Example 4.** A company manufactures and sells $x$ videophones per week. The weekly price-demand and cost equations are, respectively,

$p=500-0.5x$ and $C\left(x\right)=20,000+135x$ for $0\leq x\leq 1000$

A) What price should the company charge for the phones and how many phones should be produced to maximize the weekly revenue? What is the maximum weekly revenue?

$$R\left(x\right)=xp=x\left(500-0.5x\right)=500x-0.5x^{2}$$

$R^{'}\left(x\right)=500-x=0$ when $x=500$

$R^{''}\left(x\right)=-1$ means the graph of the revenue function is always concave down.

The company should produce 500 videophones per week and sell them at $250 apiece in order to maximize the revenue at $125,000 per week.

B) What price should the company charge for the phones and how many phones should be produced to maximize the weekly profit? What is the maximum weekly profit?

$$P\left(x\right)=R\left(x\right)-C\left(x\right)=365x-0.5x^{2}-20,000$$

$P^{'}\left(x\right)=365-x=0$ when $x=365$

$P^{''}\left(x\right)=-1$ means the graph of the profit function is always concave down.

The company should produce 365 videophones per week and sell them at $317.50 each (from price-demand equation above) in order to maximize the profit at $46,612.50 per week.