# Chapter 10 – 4a The Derivative

**Example 1.** If a ball is dropped from a height on the moon, the distance the ball will have fallen after x seconds is approximately 2.66x2 feet.

How far will the ball fall between x = 4 and x = 6? f(6) - f(4) = 95.76 – 42.56 = 53.2 ft

What is the average speed during that time interval? (f(6) – f(4))/(6 – 4) = 53.2/2 = 26.6 ft/s

How far will the ball fall between x = 4 and x = 5? f(5) - f(4) = 66.50 – 42.56 = 23.94 ft

What is the average speed during that time interval? (f(5) – f(4))/(5 – 4) = 23.94/1 = 23.94 ft/s

How far will the ball fall between x = 4 and x = 4.5? f(4.5) - f(4) = 53.865 – 42.56 = 11.305 ft

What is the average speed during that time interval? (f(4.5) – f(4))/(4.5 – 4) = 11.304/0.5 = 22.61 ft/s

How far will the ball fall between x = 4 and x = 4.1? f(4.1) - f(4) = 44.7146 – 42.56 = 2.1546 ft

What is the average speed during that time interval? (f(4.1) – f(4))/(4.1 – 4) = 2.1546/0.1 = 21.546 ft/s

What is the speed of the ball at x = 4?

$$\lim\_{h\to 0}\frac{f\left(4+h\right)-f\left(4\right)}{h}=\frac{2.66×\left(4+h\right)^{2}-2.66\left(4\right)^{2}}{h}=2.66\left(8\right)=21.28 ft/s$$

$$\lim\_{h\to 0}\frac{f\left(x+h\right)-f\left(x\right)}{h}=\frac{2.66×\left(x+h\right)^{2}-2.66\left(x\right)^{2}}{h}=2.66\left(2x\right)=5.32x ft/s$$

### Average Rate of Change

The average rate of change in the value of f(x) with respect to x between x = a and x = a+h is given by

$$\frac{f\left(a+h\right)-f\left(a\right)}{\left(a+h\right)-a}=\frac{f\left(a+h\right)-f\left(a\right)}{h}, h\ne 0$$

“Rate of change” is a general term. In some situations, a more specific term might be used. For example, if f(x) represents distance and x represents time (as in our example), the rate of change of f(x) with respect to x is called speed.

### Instantaneous Rate of Change (The Derivative)

The instantaneous rate of change in the value of f(x) with respect to x at x = a is called the derivative of f(x) at x = a, denoted f’(a), and is given by

$$\lim\_{x\to a}\frac{f\left(a+h\right)-f\left(a\right)}{\left(a+h\right)-a}=\lim\_{x\to a}\frac{f\left(a+h\right)-f\left(a\right)}{h}, if this limit exists$$

The derivative of f(x) at x = a is the slope of the line tangent to the graph of f(x) at x = a.

**Example 2.** The U.S. production of zinc (in thousands of metric tons) is approximately

$$p\left(t\right)=14t^{2}-6.6t+602.4$$

where t is the number of years since 1995 (i.e., t = 0 corresponds to 1995). (The use of t as the number of years since 1995 simplifies the equation. If t represented the year, the equation would be $p\left(t\right)=14t^{2}-55,866.6t+55,734,119.4$)

What is the average rate of change in the production of zinc between 2004 (t=9) and 2008 (t=13)?

[p(13) – p(9)]/(13 – 9) = 1205.6/4 = 301.4 thousand metric tons per year

What is the average rate of change in the production of zinc between 2004 (t=9) and 2007 (t=12)?

[p(12) – p(9)]/(12 - 9) = 862.2/3 = 287.4 thousand metric tons per year

What is the average rate of change in the production of zinc between 2004 (t=9) and 2006 (t=11)?

[p(11) – p(9)]/(11 - 9) = 546.8/2 = 273.4 thousand metric tons per year

What is the average rate of change in the production of zinc between 2004 (t=9) and 2005 (t=10)?

[p(10) – p(9)]/(10 – 9) = 259.4/1 = 259.4 thousand metric tons per year

What is the instantaneous rate of change in production in 2004 (t=9)? In 2010 (t=15)?

$$p^{'}\left(t\right)=\lim\_{h\to 0}\frac{p\left(t+h\right)-p(t)}{h}=\lim\_{h\to 0}\frac{\left[14\left(t+h\right)^{2}-6.6\left(t+h\right)+602.4\right]-\left[14t^{2}-6.6t+602.4\right]}{h}=28t-6.6$$

$p'\left(9\right)=28\left(9\right)-6.6=245.4$ thousand metric tons per year. In 2004, zinc production is increasing at the rate of 245,400 metric tons per year.

$p'\left(15\right)=28\left(15\right)-6.6=413.4$ thousand metric tons per year. In 2010, zinc production is increasing at the rate of 413,400 metric tons per year.

 

The graph on the right is better because it shows the year on the horizontal scale rather than the number of years since 1995. As a result, it is more easily interpreted. This is easy to do in Excel. You already need a t and p(t) column. Just create a “year” column where the year is t+1995. Zinc production p(t) is calculated using the t values but plotted using the year values.