# Chapter 10 – 5a Basic Differentiation Properties

$$Constant function rule: \frac{d}{dx}C=0$$

$$Constant multiple rule: \frac{d}{dx}\left(C\*f\left(x\right)\right)=C\*\frac{d}{dx}f\left(x\right)$$

$$Addition rule: \frac{d}{dx}\left(f\left(x\right)\pm g\left(x\right)\right)=\frac{d}{dx}f\left(x\right)\pm \frac{d}{dx}g\left(x\right)$$

$$Power rule: \frac{d}{dx}x^{n}=nx^{n-1}$$

**Example 1.** The estimated total sales of a company (in millions of dollars) t months from now is given by

$$S\left(t\right)=0.03t^{3}+0.5t^{2}+2t+3$$

A) Find S’(t). $S^{'}\left(t\right)=0.03\left(3t^{2}\right)+0.5\left(2t^{1}\right)+2\left(1t^{0}\right)+0=0.09t^{2}+t+2$

B) Find S(5) and S’(5) and interpret the results. S(5) = 29.25 and S’(5) = 9.25. At the end of 5 months, the total in sales is estimated to be 29.25 million dollars and increasing at the rate of 9.25 million dollars per month.

C) Find S(10) and S’(10) and interpret the results. S(10) = 103 and S’(10) = 21. At the end of 10 months, the total in sales is estimated to be 103 million dollars and increasing at the rate of 21 million dollars per month.

**Example 2.** From past records, it is estimated that the number of power boats a marine manufacturer can sell depends, to some extent, on the amount of money it spends on advertising. In particular, if x represents the advertising expenditure (in thousands of dollars), the number of power boats the manufacturer expects to sell is given by $N\left(x\right)=1000-\frac{3780}{x}=1000-3780x^{-1}$

A) Sketch the graph of this function on the interval (0, 30]. Identify any asymptotes. There is a vertical asymptote at x = 0 and horizontal asymptote at y = 1000.

B) Find N’(x). $N'(x)=0-3780\left(-1x^{-2}\right)= \frac{3780}{x^{2}}$

C) Find N(10) and N’(10) and interpret the results. N(10) = 622 and N’(10) = 37.8. If $10,000 is spent on advertising, the expected sales are 622 boats and increasing at a rate of 37.8 boats per thousand dollars of advertising.

D) Find N(20) and N’(20) and interpret the results. N(20) = 811 and N’(20) = 9.45. If $20,000 is spent on advertising, the expected sales are 811 boats and increasing at a rate of 9.45 boats per thousand dollars of advertising.

**Example 3.** If a person is x inches tall, the person’s approximate pulse rate y (beats per minute) is given by

$$y=\frac{590}{\sqrt{x}}=590x^{-\frac{1}{2}}$$

A) Sketch the graph of this function.

B) Find dy/dx.

$$\frac{dy}{dx}=590\left(-\frac{1}{2}x^{-\frac{3}{2}}\right)-\frac{295}{x\sqrt{x}}$$

C) Find y and dy/dx for x = 66 and interpret the results.

y ≈72.6 and dy/dx ≈ -0.55. A person 66 inches tall would have a pulse rate of approximately 72.6 beats per minute and the pulse rate is decreasing by about 0.55 beats per minute per inch of increased height.

D) Find y and dy/dx for x =72 and interpret the results.

y ≈69.5 and dy/dx ≈ -0.48. A person 72 inches tall would have a pulse rate of approximately 69.5 beats per minute and the pulse rate is decreasing by about 0.48 beats per minute per inch of increased height.

**Example 4.** Suppose that a person can memorize y facts in x hours as given by

$y=50\sqrt{x}=50x^{\frac{1}{2}}$

A) Sketch the graph of this function.

B) Find y’.

$$y^{'}=50\left(\frac{1}{2}x^{-\frac{1}{2}}\right)=25\frac{1}{x^{\frac{1}{2}}}=\frac{25}{\sqrt{x}}$$

C) How many facts can this person memorize in the 4th hour? What is the average rate of memorizing facts during the 4th hour? What is the instantaneous rate at the beginning and at the end of the 4th hour?

In the fourth hour, the person can memorize about 13.4 facts ($50\sqrt{4}-50\sqrt{3}$) which represents an average of 13.4 facts per hour. The instantaneous rate at the beginning of the 4th hour is about 14.4 facts per hour ($25/\sqrt{3}$). The instantaneous rate at the end of the fourth hour is about 12.5 facts per hour ($25/\sqrt{4}$).