# Chapter 10 – 6a Differentials

## Increments

An increment is a change. The notation ∆x (“delta x”) refers to the change in x and the notation ∆y (“delta y”) refers to the corresponding change in y. Consider the function y = f(x) at x1 and x2:

**Example 1.** For , find ∆x, ∆y, and ∆y/∆x for x1 = 1 and x2 = 3.

**Example 2.** For , find ∆x, ∆y, and ∆y/∆x for x1 = 1 and x2 = 3.

## Increments and Differentials

Earlier, we represented the slope of a secant line to a curve as where h represented the change in x. Since h and ∆x represent the same thing, the slope of the secant line can also be written as

Consequently, the derivative of a function can also be defined as follows:

The quotient ∆y/∆x represents the slope of the secant and the quotient dy/dx represents the slope of the tangent line. The numerator of the second quotient, dy, is called the differential of y and the denominator, dx, is called the differential of x. In practice, dx and ∆x are considered equal though it is important to understand that dy and ∆y are not, as illustrated here:



The differential of y can be calculated as shown here:

For small values of ∆x (or dx), ∆y ≈ dy so ∆y ≈ f’(x)dx. That is, we can use the differential of y to approximate the increment in y. In general, the smaller the differential of x, the more accurate this approximation will be.

**Example 3.** Find a general equation for dy and for ∆y for . Use your equations to find dy and ∆y for x = 3 and ∆x = 0.05.

Note:

 and

For x = 3 and ∆x = 2, and

**Example 4.** Find dy and ∆y for when x = -2 and ∆x = 1 and when x = 2 and ∆x = 1.

When x = -2 and ∆x = 1:

When x = 2 and ∆x = 1: