# Chapter 10 – 7a Marginal Analysis

In economics, the word “marginal” refers to an instantaneous rate of change (i.e., a derivative). Marginal cost is the instantaneous change in the total cost relative to the number of units produced. If C(x) is the total cost to produce x items, then C’(x) is the marginal cost. The marginal cost at a given value of x is an estimate of how much it will cost to produce the next item (item x+1). This estimate is the differential of y (dy) for dx = 1. The actual cost to product the next item is C(x+1)-C(x). This is the same as the increment in y (Δy) for Δx = 1.

Similarly, marginal revenue is the change in the total revenue relative to the number of items sold and marginal profit is the change in the total profit relative to the number of items sold. The marginal revenue at x is approximately equal to the revenue generated by selling the next item. The marginal profit is approximately equal to the profit generated by selling the next item. If R(x) is the total revenue function then

 and

**Example 1.** The total cost (in dollars) of producing x food processors is

A) Find the exact cost of producing the 21st food processor.

B) Use the marginal cost to estimate the cost of producing the 21st food processor.



**Example 2.** The total cost (in dollars) of manufacturing x auto body frames is

A) Find the average cost per frame if 500 frames are produced.

B) Find the marginal average cost per frame at a production level of 500 frames and interpret the results.

The average cost is decreasing at the rate of 24¢ a frame.

C) Use the results from A and B to approximate the average cost per frame if 501 frames are produced. Compare your estimate to the actual average cost.

The approximate average cost is 420.00 – 0.24 = $419.76. The actual value is $419.7605.

**Example 3.** The total profit (in dollars) from the sale of x skateboards is



A) Find the approximate profit from the sale of the 25th skateboard and compare it to the actual profit.

Approximate: Actual:

B) Find the approximate profit from the sale of the 90th skateboard and compare it to the actual profit.

Approximate: Actual:

**Example 4.** The total profit (in dollars) from the sale of x video DVDs is

A) Evaluate the marginal profit at x = 450 and interpret the results.

 The total profit is increasing at the rate of 50¢ per DVD

B) Evaluate the marginal profit at x = 750 and interpret the results.

 The total profit is decreasing at the rate of $2.50 per DVD.

**Example 5.** The demand (x units) for a clock radio depends on the price (p, in dollars) as given by

A) Find the price as a function of demand. What is the domain of this function?

Solving the demand function for price:

B) Find the revenue, R(x), from the sale of x clock radios. What is the domain of this function?

C) Find the marginal revenue at a production level of 1,600 clock radios and interpret the results.

 Total revenue is increasing at the rate of $20 per clock radio.

D) Find the marginal revenue at a production level of 2,500 clock radios and interpret the results.

 Total revenue is decreasing at the rate of $25 per clock radio.