# Chapter 10 – 7b Marginal Analysis

**Example 6.** The price-demand equation and the cost function for the production of table saws are given, respectively, by

where x is the number of saws that can be sold at a price of p dollars per saw and C(x) is the total cost of producing x saws.

A) Find the marginal cost.

B) Find the marginal average cost.

C) Find the marginal cost and the marginal average cost at a production level of 2,000 saws and interpret the results.

At a production level of 2,000 saws, the total cost is increasing at the rate of $60 per saw while the average cost per saw is decreasing at the rate of 1.8 cents per saw.

D) Find the revenue function and determine its domain.

From the price-demand equation,

E) Find the marginal revenue.

F) Find the marginal revenue for x = 1,500 and x = 4,500 and interpret the results

G) Graph the cost function and the revenue function on the same coordinate system. Find the break-even points and indicate regions of profit and loss.

The break-even points are at x=600 and x=3600. Sales between 600 and 3600 (exclusive) will generate a profit. Sales less than 600 or greater than 3600 will generate a loss.

H) Find the profit function.

I) Find the marginal profit.

J) Find the marginal profit for x = 1,500 and x = 3,000 and interpret the results.

, so the total profit is increasing at the rate of $40 per saw.

, so the total profit is decreasing at the rate of $60 per saw.

K) How many units should be sold to maximize the total profit?

The maximum profit is realized at the point where the marginal profit is zero (i.e., where the profit stops increasing and starts decreasing):

, so the maximum profit is P(2100) = $75,000.

**Example 7.** A company is planning to manufacture and market a new two-slice electric toaster. After conducting extensive market surveys, the research department provides the following estimates: a weekly demand of 200 toasters at a price of $16 per toaster and a weekly demand of 300 toasters at a price of $14 per toaster. The financial department estimates that weekly fixed costs will be $1,400 and variable costs (cost per toaster) will be $4.

A) Assuming the relationship between price and demand is linear, find both the demand as a function of price and the price as a function of demand.



B) Plot the revenue and cost functions on the same coordinate system. Identify break-even points and regions of loss and profit.

The break-even points are at x = 100 and x = 700. Sales between 100 and 700 units (exclusive) will result in a profit. Sales below 100 or above 700 will result in a loss.

C) Plot the graph of the profit function. Find the marginal profit at x = 250 and x = 475 and interpret the results.



. At a sales level of 250 toasters, the profit is increasing at the rate of $6.00 per toaster.

. At a sales level of 475 toasters, the profit is decreasing at the rate of $3.00 per toaster.

D) How many toasters should be sold to maximize the total profit?

The maximum profit will be realized when the marginal profit is zero (when the profit stops increasing and starts decreasing).

**Example 8.** The price-demand equation and the cost function for the production of garden hoses are given, respectively, by

where x is the number of garden hoses sold per week, p is the price in dollars, and C(x) is the total cost to product x garden hoses.

A) Graph the revenue and cost functions on the same coordinate system, find the break-even points, and indicate the regions of profit and loss.

The break-even points are at x = 44 and x = 258 (rounded to the nearest integer). Sales of 44 to 257 (inclusive) units will result in a profit. Sales of 43 or fewer units or 258 or more units will result in a loss.

B) How many garden hoses need to be made to maximize the profit and what is the maximum profit?