# Chapter 11 – 5 Implicit Differentiation

**Example 1.** Find the slope of the tangent to the unit circle at the point$ \left(^{\sqrt{3}}/\_{2}, ^{1}/\_{2}\right)$.



The equation for a unit circle is $x^{2}+y^{2}=1.$ We can solve this equation for y by subtracting $x^{2}$ from both sides and taking the square root:

$$y=\pm \sqrt{1-x^{2}}$$

The upper half of the circle is described by the equation

$$y=\sqrt{1-x^{2}}=\left(1-x^{2}\right)^{\frac{1}{2}}$$

$$\frac{dy}{dx}=\frac{1}{2}\left(1-x^{2}\right)^{-\frac{1}{2}}\left(-2x\right)=\frac{-x}{\sqrt{1-x^{2}}}=-\frac{x}{y}$$

At the point $\left(^{\sqrt{3}}/\_{2}, ^{1}/\_{2}\right)$, $\frac{dy}{dx}=-\frac{x}{y}=-\frac{^{\sqrt{3}}/\_{2}}{^{1}/\_{2}}=-\sqrt{3}$

**Example 2.** Find the slope of the tangent to the unit circle at the point $\left(^{\sqrt{3}}/\_{2}, ^{1}/\_{2}\right)$ using implicit differentiation.

The equation $y=\sqrt{1-x^{2}}$ represents an explicit function. The value of y is explicitly defined as a function of x:

 $y=f\left(x\right)=\sqrt{1-x^{2}}$

The equation $x^{2}+y^{2}=1$ represents an implicit function. That is, the equation implies that y is a function of x even though it doesn’t explicitly identify what that function is. We had to do some algebra to determine the explicit function.

Implicit differentiation refers to the process of finding the derivative of one variable relative to the other in an implicit function. The process of finding the derivative is the same but you have to be careful to remember that the second variable is implicitly a function of the first which means you’ll need to apply the chain rule. Taking the derivative of each side with respect to x we get:

$$2x+2y\frac{dy}{dx}=0⟹2y\frac{dy}{dx}=-2x⟹\frac{dy}{dx}=-\frac{x}{y}$$

Notice that we got the same result as before but with a lot less work.

**Example 3.** Find $\frac{dy}{dx}$ if $3x+5y+9=0$

Explicit differentiation: $y=-\frac{3}{5}x-\frac{9}{5}⟹\frac{dy}{dx}=-\frac{3}{5}$

Implicit differentiation: $3+5\frac{dy}{dx}+0=0⟹\frac{dy}{dx}=-\frac{3}{5}$

**Example 4.** Find $\frac{dy}{dx}$ if $3x^{2}-4y-18=0$

Explicit differentiation: $y=\frac{3}{4}x^{2}-\frac{9}{2}⟹\frac{dy}{dx}=\frac{3}{2}x$

Implicit differentiation: $6x-4\frac{dy}{dx}-0=0⟹\frac{dy}{dx}=\frac{3}{2}x$

**Example 5.** Find $\frac{dy}{dx}$ at (2, 1) if $x^{2}-y^{3}-3=0 $

Explicit differentiation: $y=\left(x^{2}-3\right)^{{1}/{3}}$

$$\frac{dy}{dx}=\frac{1}{3}\left(x^{2}-3\right)^{-{2}/{3}}\left(2x\right)=\frac{2x}{3\left(x^{2}-3\right)^{{2}/{3}}}=\frac{2x}{3y^{2}}$$

Implicit differentiation: $2x-3y^{2}\frac{dy}{dx}-0=0⟹\frac{dy}{dx}=\frac{2x}{3y^{2}}$

At (2, 1), $\frac{dy}{dx}=\frac{2∙2}{3∙1^{2}}=\frac{4}{3}$



**Example 6.** Find $\frac{dy}{dx}$ at (2, 3) if $xy-6=0$

Explicit differentiation: $y=\frac{6}{x}$

$$\frac{dy}{dx}=-\frac{6}{x^{2}}=-\frac{1}{x}\frac{6}{x}=-\frac{1}{x}y=-\frac{y}{x}$$

Implicit differentiation: $x\frac{dy}{dx}+y-0=0⟹\frac{dy}{dx}=-\frac{y}{x}$

At (2, 3), $\frac{dy}{dx}=-\frac{y}{x}=-\frac{3}{2}=-1.5$

**Example 7.** Find $\frac{dy}{dx}$ at (1, 0) if $e^{y}=x^{2}+y^{2}$

Implicit differentiation: $e^{y}\frac{dy}{dx}=2x+2y\frac{dy}{dx}⟹\frac{dy}{dx}=\frac{2x}{e^{y}-2y}$

At (1, 0), $\frac{dy}{dx}=\frac{2x}{e^{y}-2y}=\frac{2∙1}{e^{0}-2∙0}=2$

**Example 8.** The demand (x) for a certain item at a price of p dollars each is estimated to be $x=\sqrt{10000-p^{2}}$. Find $\frac{dp}{dx}$ using implicit differentiation.

$$1=\frac{1}{2}\left(10000-p^{2}\right)^{-\frac{1}{2}}∙-2p\frac{dp}{dx}$$

$$1=\frac{-p}{\sqrt{10000-p^{2}}}\frac{dp}{dx}$$

$$\frac{dp}{dx}=\frac{1}{\frac{-p}{\sqrt{10000-p^{2}}}}=\frac{\sqrt{10000-p^{2}}}{-p}=-\frac{x}{p}$$

Explict differentiation:

$$x=\sqrt{10000-p^{2}}⟹x^{2}=10000-p^{2}⟹p^{2}=10000-x^{2}⟹p=\sqrt{10000-x^{2}}$$

$$\frac{dp}{dx}=\frac{1}{2}\left(10000-x^{2}\right)^{-{1}/{2}}\left(-2x\right)=\frac{-x}{\sqrt{10000-x^{2}}}=-\frac{x}{p}$$