# Chapter 11 – 6 Related Rates

**Example 1.** An object is moving along a circular path (centered on the origin) at a constant speed. If the radius of the circle is 93,000,000 miles and it takes 365 days to go around the circle, how fast is the object moving? When the object is at the point (83063786, 41825919), the rate at which the x-coordinate is changing is 30,000 miles per hour. What is the rate of change in the y-coordinate?

$$speed=\frac{distance}{time}=\frac{2π\left(93,000,000 miles\right)}{365 days∙\frac{24 hours}{day}}≈66,705 mph$$

$x^{2}+y^{2}=\left(93,000,000\right)^{2}$ where x and y are both implicit functions of time t.

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0 ⟹ \frac{dy}{dt}=-\frac{x}{y}\frac{dx}{dt}=-\frac{83,063,786}{41,825,919}∙30,000≈-59,578 mph$$

**Example 2.** A boat is being pulled toward a dock by a man standing on the dock pulling the rope attached to the front of the boat. The man’s hands are 5 feet above the point where the rope is attached to the boat. If the man is hauling the rope in at 3 feet per second, how fast is the boat moving when it is 40 feet from the dock and when it is 10 feet from the dock?

Let L be the length of the rope (in feet) from the boat to the man’s hands. Let x be the distance from the front of the boat to the dock.

$$x^{2}+5^{2}=L^{2} ⟹ 2x\frac{dx}{dt}+0=2L\frac{dL}{dt} ⟹ \frac{dx}{dt}=\frac{L}{x}\frac{dL}{dt}$$

$$x=40 ⟹ L=\sqrt{1600+25}=\sqrt{1625} ⟹ \frac{dx}{dt}=\frac{\sqrt{1625}}{40}∙-3≈-3.023 ft/sec$$

$$x=10 ⟹ L=\sqrt{100+25}=\sqrt{125} ⟹ \frac{dx}{dt}=\frac{\sqrt{125}}{10}∙-3≈-3.354 ft/sec$$

**Example 3.** A rock is thrown into a still pond causing a circular ripple. If the radius of the circle is changing at the rate of 2 ft/sec, how fast is the area of the circle changing when the radius is 5 feet and when the radius is 10 feet?

$$A=πr^{2} ⟹ \frac{dA}{dt}=2πr\frac{dr}{dt}$$

$$r=5 ⟹ \frac{dA}{dt}=2π\left(5\right)\left(2\right)≈62.83 {ft^{2}}/{sec}$$

$$r=10 ⟹ \frac{dA}{dt}=2π\left(10\right)\left(2\right)≈125.66 {ft^{2}}/{sec}$$

**Example 4.** As a spherical balloon is being blown up, its radius is increasing at the rate of 3 centimeters per min. How fast is the volume changing when the radius is 5 centimeters and when the radius is 10 centimeters?

$$V=\frac{4}{3}πr^{3} ⟹ \frac{dV}{dt}=4πr^{2}\frac{dr}{dt}$$

$$r=5 ⟹ \frac{dV}{dt}=4π\left(5^{2}\right)\left(3\right)≈942.5 {cm^{3}}/{min}$$

$$r=10 ⟹ \frac{dV}{dt}=4π\left(10^{2}\right)\left(3\right)≈3,769.9 {cm^{3}}/{min}$$

**Example 5.** Boyle’s Law for enclosed gases states that if the volume is kept constant, temperature (in degrees Fahrenheit) and pressure (in pounds per square inch, or psi) is given by:

$$\frac{P}{T+460}=k$$

where k is a constant. Suppose an automobile tire has a pressure of 32 psi at 75°F and the temperature is increasing at the rate of 0.5°F per minute. What is the rate of change in the air pressure?

$$\frac{P}{T+460}=k ⟹ \left(T+460\right)\frac{dP}{dt}-P\frac{dT}{dt}=0 ⟹ \frac{dP}{dt}=\frac{P}{T+460}\frac{dT}{dt}$$

$$P=32, T=75, \frac{dT}{dt}=0.5 ⟹ \frac{dP}{dt}=\frac{32}{75+460}0.5≈0.03 psi/min$$