# Chapter 11 – 7a Elasticity of Demand

**Example 1.** A medical researcher is studying two treatments for a life-threatening illness. Using treatment A, patients are dying at the rate of 10 per week in the 11th week of treatment. Using treatment B, the death rate is 20 patients per week in the 11th week. Which treatment is better?

We don’t have enough information to answer the question. In particular, we don’t how many patients are receiving treatment A and how many are receiving treatment B. In this context, it is not the number of people who die per week that is important. It is the number of people who die compared to the number of people receiving treatment. This is called the **relative rate of change**:

The **percentage rate of change** of a function is 100% times the relative rate of change.

In the context of our example, f(x) represents the number of patients who survive the disease. Suppose there are 100 patients receiving treatment A at the end of 10 weeks. If 10 of them die during the 11th week, the relative rate of change is -10/100 or -0.1 or -10%. In practice, the researcher might say that the death rate was 10% or that the death rate was 100 patients per 1,000 patients per week.

Now suppose 300 people are receiving treatment B at the end of 10 weeks. If 20 of those patients die during the 11th week, the relative rate of change is -20/300 or about .067 or -6.7%. In practice the researcher might say the death rate was 6.7% or 67 patients per 1,000 patients per week.

**Example 2.** In a medical study, an experimental treatment is given to patients with a life-threatening illness. The study began with 400 patients and the number of patients still alive after x weeks is given by

A) Find the relative rate of change and the percentage rate of change at x = 5 weeks and interpret the results.

. The number of survivors is decreasing at the rate of 6.67 per hundred patients per week. Put another way, the death rate is 6.67%.

B) Find the relative rate of change and the percentage rate of change at x = 40 weeks and interpret the results.

. The number of survivors is decreasing at the rate of 2 per hundred patients per week. Put another way, the death rate is 2.0%.

**Example 3.** Use the price-demand equation below to determine whether demand is elastic, is inelastic, or has unit elasticity at p = 10, p = 20, and p = 30.

Given a price-demand function x = f(p), the elasticity of demand is the negative of the ratio of the relative rate of change in demand divided by the relative rate of change in price:

Generally, the slope of the function is negative since an increase in price results in a decrease in demand. Consequently, elasticity of demand is a positive number.

At a price of $10, the relative rate of change in demand is much smaller than the relative rate of change in price. For example, if the price were to increase by 10% to $11.00, the demand would only decrease by about 1.818% from 11,000 to about 10,800 (the actual demand would be 10,790). As a result, the demand is not affected very much by changes in price. We say the demand is inelastic. In general, if the elasticity of demand is less than one, the demand is said to be inelastic.

The relative rate of change in demand is the same as the relative rate of change in price. This is called unit elasticity. For example, if the price were to increase by 10% to $22.00, the demand would decrease by about 10% from 8000 to 7,200 (the actual demand would be 7,160).

The relative rate of change in demand is higher than the relative rate of change in price. For example, if the price were to decrease by 10% to $27.00, the demand would increase by about 60% to 4,800 (the actual demand would be 4,710). As a result, a change in the price will significantly affect the demand. When the demand elasticity is greater than one, the demand is said to be elastic.

**Example 4.** Given the price demand equation

A) Express the demand x as a function of the price p.

B) Find the elasticity of demand, E(p).

C) What is the elasticity of demand when p = $10? If the price is increased to $11 dollars what is the approximate change in demand?

The demand is inelastic at this price point. Increasing the price by $1 is a 10% increase. The demand would decrease by approximately 5% (from 4,000 units to 3,800 units). The actual demand would be units. Since the price-demand equation is linear, the secant line and the tangent line coincide so the estimated change in demand is, in fact, the actual change.

D) What is the elasticity of demand when p = $25? If the price is decreased to $23 dollars what is the approximate change in demand?

The demand is elastic at this price point. Decreasing the price $2 is an 8% change. The demand would increase by approximately 40% (from 1,000 to 1,400 units). The actual demand would be units.

E) What is the elasticity of demand when p = $15? If the price is increased by 5% what is the approximate change in demand?

The demand has unit elasticity at this price point. If the price is increased by 5%, the demand would decrease by approximately 5%. That is, if the price increased to $15.75, the demand would decrease by about 5% (from 3,000 to 2,850 units). The actual demand would be units.