# Chapter 13 – 4 The Definite Integral

### Approximating Areas Using Left and Right Sums

**Example 1.** Find the approximate area between the graph of y = 2x and the x-axis from x = 2 to x = 4. (The area is 12.)

We can approximate the area by finding the sum of the areas of vertical rectangles where each rectangle has the same base. The altitude of each rectangle can be the function value calculated for the x-coordinate of the left end of its base, the right end of its base, or the middle of its base (respectively):

  

Left Sum

Right Sum

Midpoint Sum

To improve the accuracy of the approximation, increase the number of rectangles. Suppose we use 8 rectangles instead of 4:

  

Left Sum

Right Sum

Midpoint Sum

**Example 2.** Find the approximate area between the graph of and the x-axis from x = 0 to x = 2. The region is a semi-circle and the actual area is .

As we did above, we will find the sum of the areas of vertical rectangles to approximate the area under the curve:

  

Left Sum

Right Sum

Midpoint Sum

Doubling the number of rectangles to 8 gives us a better approximation:

  

Left Sum

Right Sum

Midpoint Sum

### Riemann Sum

The left sum, the right sum and the midpoint sum are all special instances of a Riemann Sum. Assume that a function f(x) is continuous on the interval [a,b]. Divide the interval [a,b] into n sub-intervals of width. These subintervals have endpoints where , and for any value of i from 1 to n. (This implies that for any i from 1 to n.) If (i.e., is in the ith subinterval) the **Riemann sum** is given by:

The **left sum** is a Riemann sum in which every (the left end of the subinterval):

The **right sum** is a Riemann sum in which every (the right end of the subinterval):

The **midpoint sum** is a Riemann sum in which every (the midpoint of the subinterval):

### The Definite Integral

If f(x) is continuous on [a,b] the **definite integral** of f(x) from a to b is the limit of any Riemann sum as n approaches infinity:

The definite integral represents the signed area between the graph of f(x) and the x-axis on the interval [a,b]. The area above the x-axis is positive and the area below the x-axis is negative.

From example 1,

From example 2,

### Properties of Definite Integrals

**Example 3.** In the illustration to the right, area A is 44.8 square units, area B is 19.2 and area C is 83.2. Find

A)

B)

C)

D)

E)

F)

G)

If , ,

, and then find

H)

I)

J)

**Example 4.** In the illustration at the right, area A is 7.52, area B is 6.39, and area C is 0.68. Find

A)

B)

C)

D)

If then find

E)

F)

Actually, 15.84 is a better approximation. Our value is a little off because of round-off errors when our areas were rounded down to two decimal places.