# Chapter 14 – 2c Applications in Business and Economics

**Example 1.** Starting at age 25, you deposit $200 at the end of every month into an IRA account for retirement. If the account pays 5% compounded continuously, how much will be in the account when you retire at age 70? How much of that amount represents interest?

The future value of a single deposit, M, is given by where is the time in years and r is the nominal annual rate. For each deposit, the number of periods the deposit will earn interest is the number of periods left until retirement. If ppy is the number of periods per year, the jth deposit will earn interest for periods or years. Consequenlty, the future value of the jth deposit is given by

If n is the number of deposits (), the total amount in the account when you retire is given by

The summation term can be expressed as follows:

This is the sum of a geometric series whose first term is and whose common ratio is. The sum of a geometric series is given by where is the first term, is the common ratio, and is the number of terms.

These results can be confirmed using an Excel worksheet to find the future value of every deposit and then finding the sum.

**Example 2.** Repeat example one treating the periodic deposits as a continuous income stream given by

where is dollars per year.

The income over a time period of is the area under the curve which can be approximated by the area of a rectangle as illustrated to the right:

This amount earns interest for the remaining time and the future value is given by:

The future value of the entire income stream is given approximately by:

This equation is a Reimann sum so the future value is the limit of this sum as n goes to infinity:

In our problem, , and :

**Example 3.** If interest is earned at the rate of 6.25% compounded continuously, find the future value after four years of the continous income stream whose flow rate is given by . Find the total interest earned.



The total interest earned is the difference between what you get out of this investment (the future value) and what you put in. What you put in is the area under the graph of the rate of flow function: